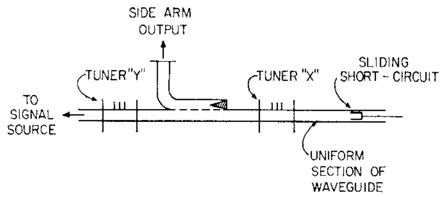

Fig. 1—A type of standard phase shifter.

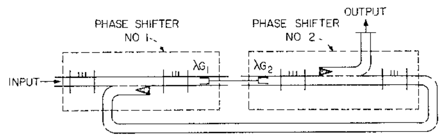

Fig. 2—Differential phase shifter.

But if one waveguide section has a different cutoff wavelength λ_c from the other, then the phase shift ψ is not equal to zero; instead,

$$\psi = 4\pi l \left(\frac{1}{\lambda_{G_2}} - \frac{1}{\lambda_{G_1}}\right) \qquad (2)$$

where the guide wavelength λ_G is related to the cutoff wavelength λ_c by

$$\frac{1}{\lambda_G^2} = \frac{1}{\lambda^2} - \frac{1}{\lambda_c^2}.$$

The cutoff wavelengths of the two waveguides can be chosen to produce any phase shift ψ between zero and the limiting case when one waveguide is operating below cutoff and the phase shift ψ is that of a single phase shifter alone.

Such a differential phase shifter has a number of potential applications such as the following. As the above standard phase shifter is extended to higher frequencies, say above 30 Gc, it takes a smaller displacement to produce the same phase shift; hence, errors in determining this displacement produce correspondingly larger errors in the phase shift. This situation can be avoided by using a differential phase shifter as described above, with the waveguide cutoff frequencies chosen so that the phase of the output varies more slowly than it would if it were tracking the position of one short circuit.

For example, at an operating frequency of 75 Gc, if one waveguide section is WR-15 and the other is WR-12, $\lambda_{G1} = 0.5230$ cm, and $\lambda_{G_2} = 0.4719$ cm. A displacement of 0.2615 cm will produce a phase shift of 39 degrees, which is approximately one ninth of the phase shift that would be produced by a single phase shifter using WR-12 waveguide.

Another application is in the investigation of uniformity of waveguide sections and the suitability of short circuits for phase shift standards. If the arrangement of Fig. 2 is used and the two waveguide sections are nominally identical, there will ideally be zero phase shift of the output as the short circuits are moved. Any phase shift which actually does occur is due to deficiencies in the short circuits or the waveguide sections, or in both.

Another application might consist of the determination of relative displacement from the measurement of phase shift. This would require that the motion of the two short circuits be independent rather than ganged. The sensitivity of phase shift to relative displacement could be preselected by choosing the cutoff frequencies of the individual waveguides as desired.

If the tuners are dispensed with, the differential phase shifter will still function but with reduced accuracy, due to the finite directivities of the directional couplers and the reflections in the system.

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An Extension of the TE_{01n} Resonator Method of Making Measurements on Solid Dielectrics

Dielectric measurements using cylindrical cavities resonating in the TE_{01n} mode are discussed by several authors.[1–4] The cavity is physically arranged as illustrated in Fig. 1, and the dielectric properties of the specimen are determined from three experimentally observed variables: the resonant frequency, the phase shift constant of the empty resonator and the Q of the cavity. The dielectric constant and loss tangent of the specimen are then calculated using formulas which are strictly valid only if the losses of the sample-filled cavity are very small.

The purpose of this communication is, first, to show that it is possible to remove the restriction of low-loss samples and, second, to suggest a departure in the experimental procedure which eliminates the necessity of performing the usual quality-factor measurements.

Fig. 1 shows a right-circular cylindrical cavity, of radius a and length l, partially filled with a disc-shaped dielectric slab of thickness d, permittivity ϵ and conductivity σ. The problem is to evaluate ϵ and σ from measured quantities that are to be specified later. As with the investigation of any resonant structure, the present study is concerned with three fundamental issues: the normal modes or free oscillations of the cavity, its forced oscillations and resonance. In what follows these issues are discussed in that order.

The normal modes of the sample-filled cavity are solutions of Maxwell's equations subject to conditions that must be satisfied at the boundaries. Thus, if the resonator has perfectly conducting walls, the general

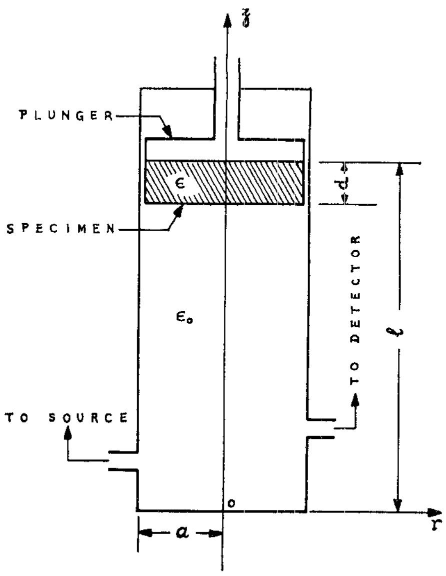

Fig. 1—TE_{01n} resonant cavity for measuring the dielectric properties of solids.

TE_{01} field configuration in the dielectric region may be specified by the familiar relations

$$\begin{aligned} E_\theta &= \frac{s\mu}{h} J_0'(hr)[Ce^{-\gamma z} + C'e^{\gamma z}]e^{st} \\ H_r &= -\frac{\gamma}{h} J_0'(hr)[Ce^{-\gamma z} - C'e^{\gamma z}]e^{st} \\ H_z &= J_0(hr)[Ce^{-\gamma z} + C'e^{\gamma z}]e^{st} \qquad (1) \\ h^2 - \gamma^2 &= -s^2\mu\epsilon_c \\ h &= 3.832/a \\ \epsilon_c &= \epsilon + \sigma/s \end{aligned}$$

where the real and imaginary parts of the frequency $s = -\omega_i + j\omega$ and those of the propagation factor $\gamma = \alpha + j\beta$ are as yet unknown. In the sample-free portion of the cavity, the corresponding field relations are

$$\begin{aligned} E_{\theta_0} &= \frac{s\mu_0}{h} J_0'(hr)[C_0e^{-\gamma_0 z} + C_0'e^{\gamma_0 z}]e^{st} \\ H_{r_0} &= -\frac{\gamma_0}{h} J_0'(hr)[C_0e^{-\gamma_0 z} - C_0'e^{\gamma_0 z}]e^{st} \\ H_{z_0} &= J_0(hr)[C_0e^{-\gamma_0 z} + C_0'e^{\gamma_0 z}]e^{st} \qquad (2) \\ h^2 - \gamma_0^2 &= -s^2\mu_0\epsilon_0 \\ h &= 3.832/a \\ \gamma_0 &= \alpha_0 + j\beta_0. \end{aligned}$$

In (2), ϵ_0 denotes the permittivity of the empty space and, except in the case of the Bessel function, the subscript zero refers to quantities pertaining to the empty portion of the sample-loaded cavity.

Boundary conditions require that the tangential component of $\boldsymbol{E}$ and the normal component of $\boldsymbol{B}$ must vanish at $z = 0$ and at $z = l$. Also, the tangential components of $\boldsymbol{E}$ and $\boldsymbol{H}$ and the normal component of $\boldsymbol{B}$ must be continuous at $z = l - d$. Applying these conditions yields

$$\frac{\mu_0}{\gamma_0} \tanh \gamma_0(l - d) + \frac{\mu}{\gamma} \tanh \gamma d = 0. \qquad (3)$$

Eq. (3) expresses mathematically the condition for free oscillations. Note that, when

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[1] A. Von Hippel, "Dielectric Materials and Applications," John Wiley and Sons, Inc., New York, N. Y., pp. 63–122; 1954.
[2] F. Horner *et al.*, "Resonance methods of dielectric measurement at centimeter wavelengths," part III, *J. Inst. Elec. Engrs.* (*London*), vol. 93, pp. 53–68; January, 1946.
[3] C. G. Montgomery, "Technique of Microwave Measurements," McGraw-Hill Book Co., Inc., New York, N. Y., p. 625; 1947.
[4] L. Hartshorn and J. A. Saxton, "The dispersion and absorption of electromagnetic waves," in "Encyclopedia of Physics," Springer-Verlag, Berlin, Germany, pp. 679–685; 1958.

$\mu=\mu_0$ and $\alpha=\alpha_0=0$ (lossless resonator), this equation reduces to

$$-\frac{\tan \beta_0(l-d)}{\beta_0}=\frac{\tan \beta d}{\beta}, \tag{4}$$

which is just (31) in Horner, *et al.*

Eq. (3) together with the relations $h^2-\gamma^2=s^2\mu\epsilon_c$ and $h^2-\gamma^2=-s^2\mu_0\epsilon_0$ impose six conditions on a total of eight field parameters, namely α_0, β_0, α, β, ω, ω_i, ϵ and σ. Solutions for any six of these may, therefore, be obtained provided that measurements have been made to evaluate the remaining two. It should be emphasized, however, that such measurements must be conducted while the sample-filled cavity is undergoing a transient response under the influence of a unit impulse of excitation. For it is then that the oscillations are natural. However, the field in the cavity may decay so rapidly that no measurements can be performed with any degree of accuracy. Of course, if the cavity losses are relatively small, as is often the case in practice, the necessary measurements can be made under steady-state sinusoidal operating conditions. It is obvious, therefore, that the equations of free oscillations are valid representations of forced oscillations only if the cavity is virtually lossless. A different set of equations is needed when the losses in the cavity are relatively high.

Careful examination of the conditions for free oscillations shows that, if the specimen under test is characterized by a finite, nonzero conductivity, the frequencies of free oscillations and the associated propagation constants are complex. The important implication of this fact is often overlooked. It is generally true that, when driven sinusoidally in time, the specimen-loaded cavity cannot be forced to oscillate at any one of its natural frequencies and that, therefore, (3) does not hold under these conditions. The significant issue in this, as in the case of any lossy resonator, is the definition of resonance. With regard to this question the viewpoint adopted here is that by resonance of a lossy system is meant the phenomenon that takes place when, under steady-state sinusoidal operating conditions, the response of the resonator reaches a relative maximum with variations in frequency. The corresponding frequencies are, by definition, the resonant frequencies of the resonator.

The next problem, therefore, is to determine the condition of resonance for the specimen-loaded cavity of Fig. 1.

It is a well-known fact that in a linear system the natural frequencies of oscillation are the poles of the transfer function for the particular problem being investigated or, stated in another way, the zeros of its denominator. Accordingly, if $D(s)$ denotes this denominator, the natural frequencies of oscillation are the roots of the algebraic equation $D(s)=0$ and the resonant frequencies may be defined by the roots of the equation

$$\frac{d}{d\omega}|D(j\omega)|=0.$$

By analogy, the *natural frequencies* of oscillation of the lossy, but linear, resonator of Fig. 1 are solutions of (3), while its *resonant frequencies* are solutions of the equation

$$\frac{d}{d\omega}(u^2+v^2)=0 \tag{5}$$

where u and v are, respectively, the real and imaginary parts of the left-hand member of (3) evaluated at $s=j\omega$, $\gamma_0=j\beta_0$ and $\gamma=\alpha+j\beta$. The condition for resonance, expressed by (5), together with the relations $h^2+\beta_0^2=\omega^2\mu_0\epsilon_0$ and $h^2-\gamma^2=\omega\mu(\epsilon-j\sigma/\omega)$ constitute a set of four equations expressing relations among a total of six variables, namely β_0, α, β, ω, ϵ and σ. The desired quantities ϵ and σ (and, hence, the loss tangent) may be determined from these equations using measured values of either ω and β, or β_0 and β. (A method for measuring phase shift constants has been reported by Simmons.[5]) The tacit assumption is, of course, that the specimen-filled cavity must be at resonance and its dimensions must remain fixed while measurements of the selected pair of variables are being made. It is evident that while the results of these measurements could ultimately be used to evaluate the Q of the specimen, the solution of the problem at hand may be completed by the present method without introducing Q into the calculations.

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[5] A. J. Simmons, "TE_{01} Mode Components in the 3mm Region," presented at the Millimeter and Submillimeter Wave Conference, Orlando, Fla.; January 7-10, 1963.

Bounds on the Elements of the Susceptance Matrix for Asymmetrical Obstacles in Waveguides

There exists a method[1-3] for the determination of upper and lower bounds on the elements of the reactance matrix B, or the equivalent network elements, for multichannel scattering. This technique was applied[4] to specific examples of lossless obstacles in a rectangular waveguide, which are symmetric with respect to some plane perpendicular to the axis of the waveguide. The problem was analyzed in terms of uncoupled even and odd standing waves. Numerical results were obtained[3] for one-dimensional quantum mechanical scattering by an asymmetric potential $V(x) \neq V(-x)$.

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[1] T. Kato, "Upper and lower bounds of scattering phases," *Progr. Theoret. Phys. (Kyoto)*, vol. 6, pp. 394–407; May, 1951.

[2] L. Spruch and R. Bartram, "Bounds on the elements of the equivalent network for scattering in waveguides. I. Theory," *J. Appl. Phys.*, vol. 31, pp. 905–913; May, 1960.

[3] R. Bartram and L. Spruch, "Bounds on elements of the S matrix for elastic scattering: One-dimensional scattering," *J. Math Phys.*, vol. 3, pp. 287–296, March–April, 1961.

[4] R. Bartram and L. Spruch, "Bounds on the elements of the equivalent network for scattering in waveguides. II. Application to dielectric obstacles," *J. Appl. Phys.*, vol. 31, pp. 913–917; May, 1960.

It is the purpose of this communication to derive bounds on nonsymmetric obstacles in rectangular waveguide (see Fig. 1) by following the procedure of Bartram and Spruch,[3] and adapting certain of their results.[2-4] (We refer the reader to the above mentioned references for a discussion of the details which are only sketched or omitted here.)

The electric field intensity $\boldsymbol{E}(\boldsymbol{r})$ satisfies the differential matrix equation

$$\mathcal{L}\boldsymbol{E}=-\nabla\times\nabla\times\boldsymbol{E}+[(\omega^2/c^2)+\boldsymbol{V}]\boldsymbol{E} = 0. \tag{1}$$

$\boldsymbol{E}$ and the matrix potential $\boldsymbol{V}$ are expressed in terms of even and odd functions of z, the direction of propagation.

$$\boldsymbol{E}=\begin{pmatrix}\mathbf{E}_e\\ \mathbf{E}_0\end{pmatrix},\qquad \boldsymbol{V}=\frac{1}{2}\begin{pmatrix}W_e & W_0\\ W_0 & W_e\end{pmatrix}, \tag{2}$$

where

$$W_e=W_0=W=\omega^2(\epsilon-1)/c^2.$$

ω, c and ϵ are the angular frequency, velocity of light and relative permittivity of the obstacle, respectively. Since the two channels (corresponding to the even and odd portions of the electric field) are coupled by the matrix potential, three parameters are required to describe the asymptotic effects of the scattering process. The asymptotic form of $\boldsymbol{E}$ for $z\to+\infty$ is

$$\boldsymbol{E}=\mathbf{f}(x,y)[\boldsymbol{e}_\theta\cos(kz+\theta) - \boldsymbol{B}_\theta\boldsymbol{e}_\theta\sin(kz+\theta)], \tag{3}$$

where $\mathbf{f}(x,y)$ is the form function for the propagating mode, $\boldsymbol{B}_\theta$ is the susceptance matrix, $\boldsymbol{e}_\theta$ is an amplitude column matrix $0\leq\theta\leq\pi$, and k^2 is $(\omega/c)^2-(\pi/a)^2$ (a is the wide dimension of the guide).

In order to obtain bounds on the susceptance matrix we have to consider an associated eigenvalue problem with certain boundary conditions,

$$\mathcal{L}\boldsymbol{\psi}_n(\mathbf{r})+\boldsymbol{\mu}_n\boldsymbol{\rho}(\mathbf{r})=0, \tag{4}$$

where $\boldsymbol{\psi}_n$ and μ_n are its eigenfunctions and eigenvalues, respectively, and where $\rho(\mathbf{r})$ is a real, positive definite Hermitian matrix. Let α_θ and $-\beta_\theta$ be the smallest positive and smallest (in absolute value) negative eigenvalue, respectively, associated with the eigenmodes of (4). The upper and lower bounds on a quadratic form of the susceptance matrix are[5]

$$\begin{aligned}-\alpha_\theta^{-1}\int(\mathcal{L}\boldsymbol{E}_t)^\dagger(\boldsymbol{\rho}^{-1}\mathcal{L}\boldsymbol{E}_t)d\tau &\leq k\boldsymbol{e}_\theta^\dagger\boldsymbol{B}_{\theta t}\boldsymbol{e}_\theta-k\boldsymbol{e}_\theta^\dagger\boldsymbol{B}_\theta\boldsymbol{e}_\theta + \int\boldsymbol{E}_t^\dagger\mathcal{L}\boldsymbol{E}_t d\tau \\ &\leq \beta_\theta^{-1}\int(\mathcal{L}\boldsymbol{E}_t)^\dagger(\boldsymbol{\rho}^{-1}\mathcal{L}\boldsymbol{E}_t)d\tau,\end{aligned} \tag{5}$$

where $\boldsymbol{E}_t$ is a trial function which is required to have the asymptotic form of $\boldsymbol{E}$, (3), but the unknown $\boldsymbol{B}_\theta$ is replaced by $\boldsymbol{B}_{\theta t}$. The range of integration of $d\tau$ is over the interior of the waveguide ($z\geq0$).

The above theory will now be applied to nonsymmetric obstacles in waveguide extending a distance d in the z direction (see

[5] The symbol † stands for the Hermitian adjoint.